Shape analysis of complex symmetric structures: Estimating components of symmetric variation and asymmetry

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1 Introduction

The method of object symmetry (Mardia et al. 2000) has been elaborated for the studies of shape variation in structures that have an internal plane of bilateral symmetry (e.g. vertebrate skulls and insect bodies). Further work related object symmetry to the structure of the shape tangent space (Kolamunnage & Kent 2003, 2005). For instance, principal component analysis (PCA) can separate a component of symmetric shape variation and components of asymmetric shape variation (Kolamunnage & Kent 2003).

The approach of Mardia et al. (2000) for the studies of shape variation in bilaterally symmetric shapes has been extended for the analysis of any type of symmetry (Savriama & Klingenberg 2006). In particular, the method of object symmetry has been generalised to any structure that exhibits more complex internal symmetry (e.g. radial symmetry in corals). Every type of symmetry is associated with a set of symmetry transformations that forms a symmetry group. For example, the identity and reflection are the symmetry transformations that characterise bilateral symmetry. The first step of the analysis is to assemble a dataset from copies of an original configuration of landmarks to which all transformations in the symmetry group of the object have been applied. Thereafter, all configurations in the dataset are superimposed in a single Procrustes fit. The resulting Procrustes mean (consensus) is symmetric.

Here we follow the approach of Kolamunnage & Kent (2003, 2005) to explore the patterns of variation in the total shape tangent space for structures with complex internal symmetry by using PCA.

2 Decomposing deviations from symmetry: Estimating components of symmetric variation and asymmetry

We simulated configurations of landmarks with different types of symmetry in two dimensions with a small amount of isotropic variation around each landmark. We present the results for two examples: a configuration of 9 landmarks with a reflection and a rotation of order 2 (example 1) and a configuration of 10 landmarks with a rotation of order 3 (example 2). We perform a PCA on the covariance matrix of the Procrustes tangent coordinates.

For the example 1, there are 14 distinct principal components (PC) (Figure 1B). Of these PCs, three are symmetric (Figure 1C), four are asymmetric relative to the reflection (Figure 1D), three are asymmetric concerning the rotation (Figure 1E), and four are asymmetric with respect to both reflection and rotation (Figure 1F). All eigenvalues are distinct.

For the example 2, there are 16 PCs, of which some are singles, whereas many others occur as pairs with equal eigenvalues (Figure 2B). Two single PCs are completely symmetric (Figure 2C) and two single PCs are rotationally symmetric but asymmetric relative to the reflection.
Figure 1: Decomposition of departures from the consensus for a configuration with a reflection and a rotation of order 2. Note that the vertical axis is the axis of reflection symmetry. The straight lines represent departures for each landmark from the consensus. A. Consensus. B. Percentages of total variance for which the PCs account. C. Example of a PC that represents a symmetric shape change. D. Example of a PC that accounts for an asymmetric shape change due to the reflection (rotational symmetry preserved). E. Example of a PC that describes an asymmetric shape change relative to the rotation (reflection symmetry preserved). F. Example of a PC that shows an asymmetric shape change relative to both reflection and rotation.
Figure 2: Decomposition of departures from the consensus for a configuration with a reflection with a rotation of order 3. Note that the vertical axis is the axis of reflection symmetry. The straight lines represent departures for each landmark from the consensus. A. Consensus. B. Percentages of total variance for which the PCs account. Note that there are single PCs and pairs of PCs. C. Example of a PC that represents a symmetric shape change. D. Example of a PC that accounts for an asymmetric shape change due to the reflection (rotational symmetry preserved). E. Example of one pair of PCs with equal eigenvalues. F. Example of a scatterplot of the scores for one pair of PCs. The pair of PCs have equal eigenvalues with opposite signs that is why the scatterplot shows a circular distribution of the scores.
(Figure 2D). There are 12 pairs of PCs and both PC of each pair, taken separately, appear asymmetric (Figure 2E), but it is a rotation of the pair of PCs that would reveal symmetry in the shape changes. A scatterplot of the scores for the pairs of PCs shows a distribution of the scores that shows rotation and reflection symmetry (Figure 2F).

The clear patterns of shape changes shown by the example 1 suggest that for any type of symmetric structure, the departures from the consensus can be decomposed into four components of shape changes: symmetry with respect to both reflection and rotation, asymmetry relative to reflection, asymmetry considering rotation, and asymmetry to both reflection and rotation. In contrast, some of the patterns of shape changes found in the example 1 do not occur in the example 2. For the example 2 and for types of symmetry with rotations with an order higher than 2 there are systematically pairs of PCs with equal eigenvalues. In this case the patterns of shape changes are more complicated to identify and some of them that have been found for the example 1 but not in the example 2 might be hidden in these pairs of PCs with equal eigenvalues.

References


