Inference for marked point processes

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1 Introduction

The area of marked point processes is well developed but their simulation is still a challenging problem when mark correlations are to be included. In this research we propose the use of simulated annealing to incorporate the spatial mark correlation into the simulations of correlated mark point processes.

2 The simulation method

We use a spatial cumulative mark correlation function defined as:

$$K_m(t) = \frac{1}{K(t)\overline{f}_m} \frac{dw_d}{\lambda^2} \int_0^1 u^{d-1} \lambda_2^f(u) du,$$

where $w_d = \sqrt{\pi^d}/\Gamma(1 + d/2)$ is the volume of the unit ball in $\mathbb{R}^d$. $K_m(t)$ is a scaled version of the $K_f(t)$ function defined in Penttinen and Stoyan (1989), where $K(t)$ is used as the scaling function so that mark correlation features will not be diluted by the K-function. In this definition, λ_2^f is the second order f-product density function by analogy with λ_2 and is defined in Stoyan and Stoyan (1994, p. 263), where f refers to the mark function $f_m(u)$. Depending on the application, different forms of function can be used for the mark function $f_m(u)$. The most commonly used is the product of the marks, i.e.

$$f_m(u) = z_i z_j,$$

where z_i and z_j are the marks at event i and j separated by distance u.

We propose the use of simulated annealing to introduce the spatial mark correlations into the simulation. The annealing process optimises the objective function Q defined as

$$Q(Z|P) = \int \frac{\left[\widehat{K}_m^i(t) - K_m(t)\right]^2}{K_m^2(t)} dt$$

using an annealing schedule given by

$$P\{\text{swap}\} = \begin{cases} 1 & \text{if } Q_{new} < Q_{old} \\ \exp\left\{\frac{Q_{new} - Q_{old}}{u}\right\} & \text{otherwise} \end{cases}$$

where $K_m(t)$ is the target spatial cumulative mark correlation function and $\widehat{K}_m^i(t)$ is the corresponding empirical spatial mark correlation function at the i^{th} annealing step.

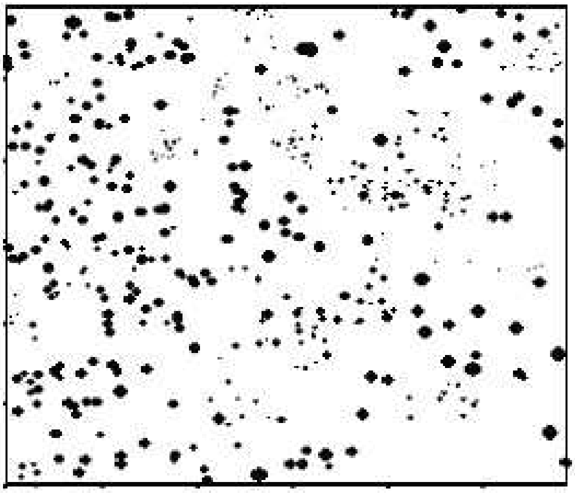

Figure 1 Forest dataset

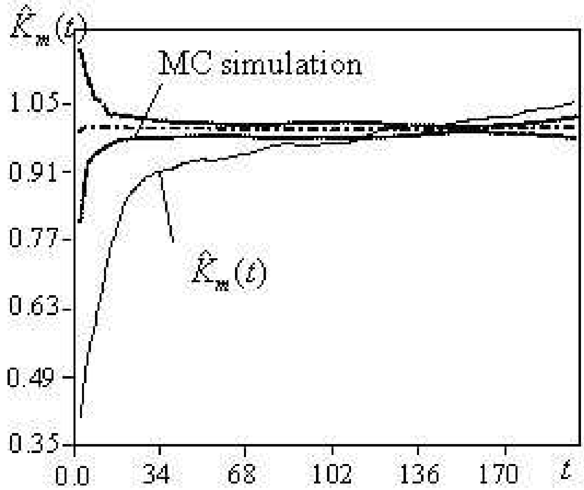

Figure 2 $\hat{K}_m(t)$ of the data

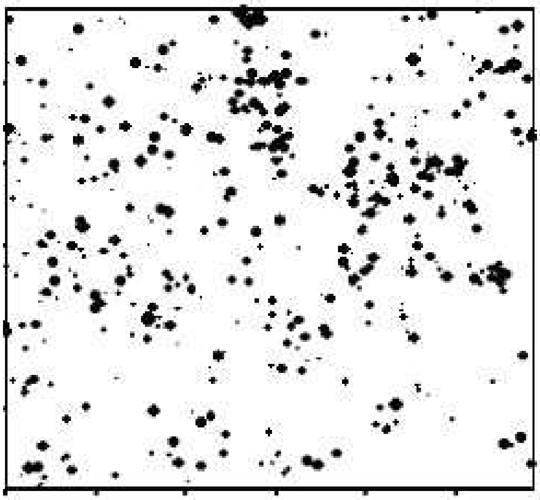

Figure 3 (a) Initial simulated pattern

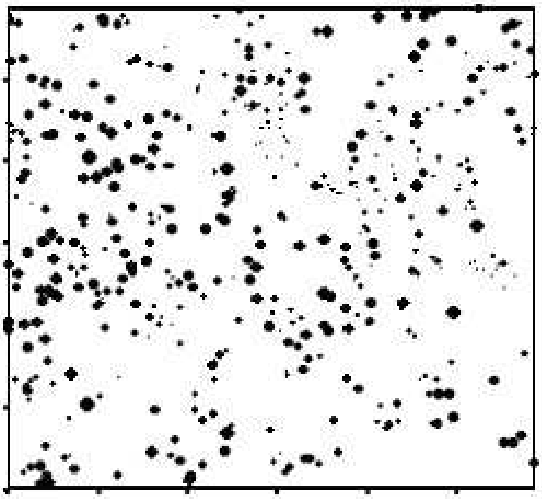

Figure 3 (b) Final simulated pattern

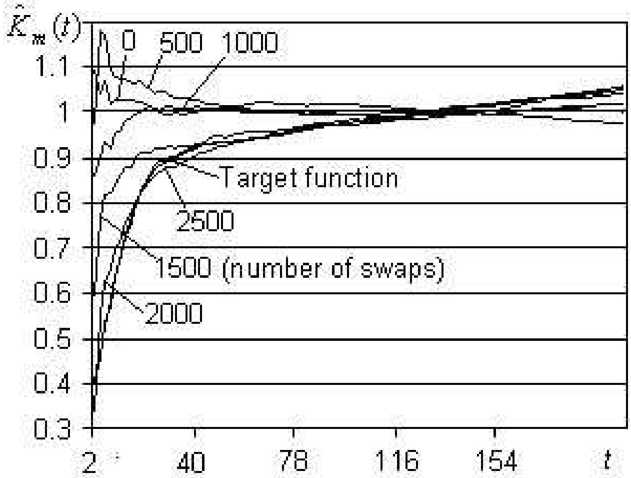

Figure 4 Annealing process

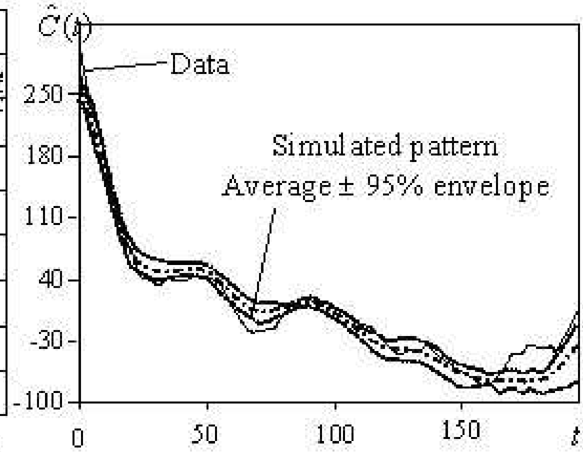

Figure 5 Mark covariance

3 Example

The method is applied to a published forest dataset (Cressie, 1993) shown in Figure 1, for which the empirical $\widehat{K}_m(t)$ function is given in Figure 2. The initial realisation of the simulation and the final simulated pattern following the annealing process are given in Figure 3. The simulated annealing process is shown in Figure 4. Characteristics of the dataset, such as point pattern summary statistics, mark correlation, mark covariance and variogram are very well reproduced. For example, Figure 5 shows a comparison of mark covariance for the dataset and the simulated pattern.

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