Simulating mark correlations in spatial point processes

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Spatial mark correlation is an important component of modelling marked point processes. The correlation function is defined by extending the definition of the K-function as follows:

\[
K_m(t) = \frac{1}{K(t)} \cdot \frac{d \cdot \sqrt{\pi^d}}{f_m \cdot \lambda \cdot \Gamma(1 + d/2)} \cdot \int_0^t ud^{d-1} \cdot \lambda(u) \cdot f_m(u) \cdot du
\]

where \( f_m(u) \) is the mark function, which is calculated using the marks at the pair of points separated by distance \( u \); \( \hat{f}_m \) is the expected value of the mark function. An example of such a function is shown in Figure 1.

![MC simulation (no correlation)](image)

\( \hat{K}_m(t) \) of data

Figure 1 \( \hat{K}_m(t) \) vs \( t \)

It is, however, difficult to incorporate this correlation function into the simulation process using the conventional approach. In this research work, we propose the use of simulated annealing to introduce the spatial auto- and cross-mark correlations into the simulation. The annealing process optimises the objective function \( Q \) defined as:

\[
Q(Z/\varphi) = \sum_N \sum_t \left( \frac{\hat{K}_m^2(t) - K_m(t)}{K_m^2(t)} \right)^2
\]

using an annealing schedule given by:

\[
P\{\text{swap}\} = \begin{cases} 
1 & \text{if } Q_{\text{new}} < Q_{\text{old}} \\
\frac{1}{e^{\frac{Q_{\text{new}} - Q_{\text{old}}}{\alpha}}} & \text{otherwise}
\end{cases}
\]

The method can also take into account the spatial correlations between marks and locations. The methods are applied to a set of simulated rock fractures and a published forest dataset (Cressie,
1993) shown in Figure 2. The initial realisation of the simulation and the final simulated pattern following the annealing process are given in Figure 3. The significant characteristics of the dataset, such as point pattern summary statistics, mark auto- and cross-correlations, and mark spatial covariances and variograms are very well reproduced. For example, Figure 4 shows a comparison of mark spatial covariances and variograms for the dataset and the simulated pattern.

References


Figure 2 (a) Forest data

Fig. 2 (b) Optimal non-parametric model ($\times 10^2$)

Figure 3 (a) Initial simulated pattern

Figure 3 (b) Final simulated pattern

Figure 4 (a) Mark variograms

Figure 4 (b) Mark covariance