The Cauchy kernel on $\mathbb{C}$ and elsewhere

Any holomorphic function on $\mathbb{C}$ explodes at infinity. In other words, there are no functions holomorphic on $\mathbb{C} \cup \{\infty\}$. In a suitable framework, this makes the Cauchy-Riemann operator invertible. Since it is invariant under conformal maps, we can conveniently map $\mathbb{C} \cup \{\infty\}$ to the sphere S^2, and inverting the Cauchy-Riemann operator leads us to the Cauchy kernel.

This setting can be extended. On an m-dimensional manifold-with-boundary M with definite metric we can generalise the Cauchy-Riemann operator to the Dirac operator (in the talk, we only use embedded manifolds to avoid vector bundles and connections, which are a bit frightening for the uninitiated). With an elegant construction (adding a so-called *collar* to M, and then duplicating the result) we obtain an object which looks sufficiently like the sphere S^2 to make the technique work. We consider some of the consequences for boundary value problems on M.

Boundary values, boundary spinors, and the Dirac operator

A classical result in complex analysis is the following: take the Bergman space of the upper half plane (i.e. the space of holomorphic functions in $\mathbb{C}^+$ with boundary values in $L_2(\mathbb{R})$). If $f = \Re f + \imath \Im f$ is such a boundary function, then $\|\Re f\|_2$ equals $\|\Im f\|_2$. Moreover, $f = \Re f - \imath \Im f$, the Hilbert transform of f, is the boundary function of a holomorphic function in $\mathbb{C}^-$. This is not true for general domains (it *almost* works for the unit disk, but you have to exclude constant functions). In this talk we generalise this result to metric manifolds-with-boundary. Obviously, this generalisation includes domains in $\mathbb{C}$, making it one of the very few discoveries in complex analysis which have been formulated first in a general setting, and were then specialised to $\mathbb{C}$.